IU Qual errata

CLASS SCHEDULE

DATE ____________________

NAME ____________________ SCHOOL ____________________

ADDRESS ____________________

PERIOD	Monday	ROOM	Tuesday	ROOM	Wednesday	ROOM	Thursday	ROOM	Friday	ROOM
1										
2										
3										
4										
5										
6										
7										
8										
9										

Apologies are not pass/fail. When giving an apology, any performance lower than an A really doesn't cut it. Proper apologies have three parts.

1. What I did was wrong
2. I feel badly that I hurt you
3. How do I make this better

— Dr. Randy Pausch (The Last Lecture)

5.

b. (2016-2017)

$$m\ddot{\varepsilon} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{q}{(a+\varepsilon)^2} - \frac{q}{(3a+\varepsilon)^2} - \frac{Q}{(4a+\varepsilon)^2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{q}{a^2}\left(1+\varepsilon/a\right)^{-2} - \frac{q}{9a^2}\left(1+\varepsilon/3a\right)^{-2} - \frac{Q}{16a^2}\left(1+\varepsilon/4a\right)^{-2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\cdot\left[\frac{-2q\varepsilon}{a^3} + \frac{2q\varepsilon}{27a^3} + \frac{2Q\varepsilon}{64a^3}\right]$$

$$= \frac{-2\varepsilon Q}{4\pi\varepsilon_0 a}\left[\frac{52q}{27a^2} - \frac{2Q}{64a^2} \cdot \frac{128q}{9}\right]$$

$$= \frac{-\varepsilon Q}{2\pi\varepsilon_0 a}\left[\frac{46q}{27a^2}\right]$$

$$= -\varepsilon\left[\frac{128q}{9\cdot 18\pi\varepsilon_0 a}\cdot\frac{46q}{27a^2}\right] = -\varepsilon\left[\frac{2944q^2}{243\pi\varepsilon_0 a^3}\right]$$

$$\omega = \sqrt{\frac{2944q^2}{243\pi\varepsilon_0 a^3 m}}$$

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{243\pi\varepsilon_0 a^3 m}{2944q^2}}$$

8. (2016-2017). I want to redo this math

$PV = k_B T \ln Z_G$

$Z_G = \dfrac{1}{1 - z\exp(-\beta\varepsilon)}$ Since Bose condensed, $z = 1$ ($\mu = 0$)

$\ln Z_G = -\ln[1 - \exp(-\beta\varepsilon)]$

$= -\dfrac{4\pi V}{h^3}\int_{-\infty}^{\infty} \ln[1-\exp(-\beta\varepsilon)]\,p^2\,dp$ $\quad 2m\varepsilon = p^2$, $m\,d\varepsilon = p\,dp$

$= -\dfrac{4\pi V}{h^3}\int^{\infty} \ln[1-\exp(-\beta\varepsilon)]\sqrt{2m\varepsilon}\cdot m\,d\varepsilon$

$= -\dfrac{4\pi V\sqrt{2}\,m^{3/2}}{h^3}\int_0^{\infty} \ln[1-\exp(-\beta\varepsilon)]\,\varepsilon^{1/2}\,d\varepsilon$

$u = \ln[1-\exp(-\beta\varepsilon)]$ $\quad v = \tfrac{2}{3}\varepsilon^{3/2}$

$du = \dfrac{1}{1-\exp(-\beta\varepsilon)}\cdot\beta\exp(-\beta\varepsilon)$ $\quad dv = \varepsilon^{1/2}$

$= -\dfrac{4\pi V\sqrt{2}\,m^{3/2}}{h^3}\left[\dfrac{2\ln[1-\exp(-\beta\varepsilon)]\varepsilon^{3/2}}{3}\Big|_0^{\infty} - \int_0^{\infty}\dfrac{2\varepsilon^{3/2}\exp(-\beta\varepsilon)\beta\,d\varepsilon}{3(1-\exp(-\beta\varepsilon))}\right]$

$= \dfrac{8\sqrt{2}\pi V m^{3/2}}{3h^3}\int_0^{\infty}\dfrac{\varepsilon^{3/2}\beta}{\exp(\beta\varepsilon)-1}\,d\varepsilon$ $\quad \beta\varepsilon = y$, $\beta\,d\varepsilon = dy$

$= \dfrac{8\sqrt{2}\pi V m^{3/2}}{3h^3}\cdot\dfrac{\beta}{\beta\cdot\beta^{3/2}}\int_0^{\infty}\dfrac{y^{3/2}}{\exp(y)-1}\,dy$

$PV = \dfrac{8\sqrt{2}\pi m^{3/2} V\cdot\Gamma(5/2)\,g_{5/2}(1)}{3h^3\beta^{5/2}}$

$P \propto \beta^{-5/2}$

10. (2016-2017)

b. Another way to do this

$\vec{p} = p_0(\hat{x} + i\hat{y})$ $\quad p_0 = 2\pi\sigma R^3/3$

$\hat{n} = \hat{r}$

$\hat{x} = \sin\theta\cos\phi\,\hat{\rho} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$

$\hat{y} = \sin\theta\sin\phi\,\hat{\rho} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

$\vec{p} = p_0[(\sin\theta\cos\phi + i\sin\theta\sin\phi)\hat{\rho} + (\cos\theta\cos\phi + i\cos\theta\sin\phi)\hat{\theta} + (-\sin\phi + i\cos\phi)\hat{\phi}]$

$\hat{n}\times\vec{p} = \begin{vmatrix}\hat{\rho} & \hat{\theta} & \hat{\phi}\\ 1 & 0 & 0\\ p_\rho & p_\theta & p_\phi\end{vmatrix} = (0, -(p_\phi), p_\theta)$

$= p_0[(\sin\phi - i\cos\phi)\hat{\theta} + (\cos\theta\cos\phi + i\cos\theta\sin\phi)\hat{\phi}]$

$|\hat{n}\times\vec{p}|^2 = p_0^2(\sin^2\phi + \cos^2\phi + \cos^2\theta\cos^2\phi + \cos^2\theta\sin^2\phi)$

$= p_0^2(1 + \cos^2\theta)$

$\dfrac{dP}{d\Omega} = \dfrac{c^2 Z_0 k^4}{8\cdot 32\pi^2}\cdot\dfrac{4\pi^2\sigma^2 R^6(1+\cos^2\theta)}{9}$

$= \dfrac{c^2 Z_0 k^4\sigma^2 R^6}{72}(1+\cos^2\theta)$

c. Polarization should be in the same direction as $\vec{E}$

$(\hat{n}\times\vec{p})\times\hat{n}$

$(\hat{n}\times\vec{p})\times\hat{n} = \begin{vmatrix}\hat{\rho} & \theta & \phi\\ 0 & -p_\phi & p_\theta\\ 1 & 0 & 0\end{vmatrix} = (0, +p_\theta, p_\phi)$

$= (0, p_\theta, p_\phi)$

d. Even though a magnetic dipole will be created, there won't be any radiation since there is no rotation.

11. (2016-2017)

a. $E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2m}\left(n_x^2/a^2 + n_y^2/b^2\right)$

$\psi = \sqrt{2/a}\sqrt{2/b}\,\sin\left(n_x\pi x/a\right)\sin\left(n_y\pi y/b\right)$

b. $E_n^{(2)} = \frac{\lambda^2 m^2}{16\pi^2\hbar^2}\cdot\frac{1}{(1/a^2+1/b^2)-(4/a^2+4/b^2)}$

$= \frac{\lambda^2 m^2}{16\pi^2\hbar^2}\cdot\frac{a^2b^2}{(a^2+b^2)-4(a^2+b^2)}$

$= \frac{-\lambda^2 m^2}{16\hbar^2\pi^2}\cdot\frac{a^2b^2}{3(a^2+b^2)}$

12. (2016-2017)

a. $V(x) = V(-x)$. Thus, the ground state wavefunction should be the same under parity

(2016-2017) 15. $P_\pi = P_\mu + P_\gamma$

$P_\mu = P_\gamma - P_\pi$

$P_\mu^2 = P_\gamma^2 + P_\pi^2 - 2P_\gamma P_\pi$

$m_\mu^2 = m_\gamma^2 + m_\pi^2 - 2(E_\gamma E_\pi - |\vec{p}_\gamma||\vec{p}_\pi|\cos\theta)$ maximized if all in the same direction

$m_\mu^2 = m_\pi^2 - 2E_\gamma\left(E_\pi - \sqrt{E_\pi^2 - m_\pi^2}\right)$

$2E_\gamma = \frac{m_\pi^2 - m_\mu^2}{E_\pi - \sqrt{E_\pi^2 - m_\pi^2}}$

$E_\gamma = \frac{m_\pi^2 - m_\mu^2}{2\left(E_\pi - \sqrt{E_\pi^2 - m_\pi^2}\right)}$

$\approx 4.27\text{ GeV}$

3. (2015-2016)

b. $\left.\frac{dV_{eff}}{dr}\right|_{r=r_0} = 0$

$\frac{-\ell^2}{mr_0^3} + m\omega^2 r_0 = 0$

$m\omega^2 r_0 = \frac{\ell^2}{mr_0^3}$

$m^2\omega^2 r_0^4 = \ell^2$

$r_0 = \left(\frac{\ell}{m\omega}\right)^{1/2}$

4. (2015-2016)

c. $\vec{J}_D = \varepsilon_0\frac{\partial\vec{E}}{\partial t} = -\frac{\varepsilon_0}{\tau^2}\cdot\frac{\mu_0 I_0\exp(-t/\tau)}{4\pi b^2}\rho^2\hat{z}$

$\vec{I}_D = \frac{-I_0 b^2}{\tau^2 c^2}\exp(-t/\tau)\,\hat{z}$

d. $\mu_0 \to \mu$, which is greater than μ_0. Then $\vec{I}_D$ should increase

13. 2015-2016

b. $\psi = \sin\left(\sqrt{2ME/\hbar^2}\cdot r + \delta_0\right)$ $\quad K_0 = \sqrt{2ME/\hbar^2}$

$\psi' = \sin\left(\sqrt{2M(E+V_0)/\hbar^2}\cdot r\right)$ $\quad K = \sqrt{2M(E+V_0)/\hbar^2}$

$\psi = \psi'$

$\frac{d\psi}{dr} = \frac{d\psi'}{dr}$

$\sin(K_0R + \delta_0) = \sin(KR)$

$K_0\cos(K_0R + \delta_0) = K\cos(KR)$

$K\tan(K_0R + \delta_0) = K_0\tan(KR)$

$\delta_0 = \tan^{-1}\left(\frac{K_0}{K}\tan(KR)\right) - K_0R$

c. $\sigma = {}^{4\pi}/_{K^2}\,\sin^2(\delta_0)$

$$= \frac{4\pi}{K_0^2}\sin^2\left[\tan^{-1}\left(\frac{K_0}{K}\tan(KR)\right) - K_0 R\right]$$

14. (2015-2016)

a. $(\hat{a}\cdot\vec{S}_1)\otimes(\hat{b}\cdot\vec{S}_2) = ((\hat{a}\cdot S_1)\otimes I)(I\otimes(\hat{b}\cdot S_2))$

$$= \begin{pmatrix} a_z & 0 & a_x - ia_y & 0 \\ 0 & a_z & 0 & a_x - ia_y \\ a_x + ia_y & 0 & -a_z & 0 \\ 0 & a_x + ia_y & 0 & -a_z \end{pmatrix}\begin{pmatrix} b_z & b_x - ib_y & 0 & 0 \\ b_x + ib_y & -b_z & 0 & 0 \\ 0 & 0 & b_z & b_x - ib_y \\ 0 & 0 & b_x + ib_y & -b_z \end{pmatrix}$$

$$= \begin{pmatrix} a_z b_z & a_z(b_x - ib_y) & b_z(a_x - ia_y) & (a_x - ia_y)(b_x - ib_y) \\ a_z(b_x + ib_y) & -a_z b_z & (a_x - ia_y)(b_x + ib_y) & -b_z(a_x - ia_y) \\ b_z(a_x + ia_y) & (a_x + ia_y)(b_x - ib_y) & -a_z b_z & -a_z(b_x - ib_y) \\ (a_x + ia_y)(b_x + ib_y) & -b_z(a_x + ia_y) & -a_z(b_x + ib_y) & a_z b_z \end{pmatrix}$$

$$\Phi = {}^1/_{\sqrt{2}}\left[\begin{pmatrix}1\\0\\0\\1\end{pmatrix} - \begin{pmatrix}0\\1\\1\\0\end{pmatrix}\right] = {}^1/_{\sqrt{2}}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}$$

$$(\hat{a}\cdot\vec{S}_1)(\hat{b}\cdot\vec{S}_2)|\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \cancel{a_z b_z - a_z(b_x - ib_y)} - \cancel{b_z(a_x - ia_y)} + (a_x - ia_y)(b_x - ib_y) \\ \cancel{a_z(b_x + ib_y) + a_z b_z} - (a_x - ia_y)(b_x + ib_y) - \cancel{b_z(a_x - ia_y)} \\ \cancel{b_z(a_x + ia_y)} - (a_x + ia_y)(b_x - ib_y) + a_z b_z - \cancel{a_z(b_x - ib_y)} \\ (a_x + ia_y)(b_x + ib_y) + \cancel{b_z(a_x + ia_y)} + \cancel{a_z(b_x + ib_y)} - a_z b_z \end{pmatrix}$$

$$\langle\Phi|(\hat{a}\cdot\vec{S}_1)(\hat{b}\cdot\vec{S}_2)|\Phi\rangle$$

$$= \frac{1}{2}\cdot\cancel{\left[-b_z(a_x - ia_y) + (a_x - ia_y)(b_x - ib_y) - a_z(b_x + ib_y) + (a_x - ia_y)(b_x + ib_y)\right.} \;\; \cancel{+b_z(a_x - ia_y)}$$

$$= \frac{1}{2}\left[a_x b_x - a_y b_y + a_x b_x + a_y b_y + a_x b_x + a_y b_y + a_x b_x - a_y b_y + 2a_z b_z\right]$$

$$= \hat{a}\cdot\hat{b}\cdot\frac{\hbar^2}{4}$$

8. (2014-2015)

b. $\langle z\rangle = \frac{-1}{\beta m}\cdot\frac{d\ln Z}{dg}$

$$= \frac{-1}{\beta m}\left[\frac{N}{A(1 - \exp(-\beta mgh))}\cdot A\beta mh\exp(-\beta mgh) - \frac{N}{\beta mg\lambda^3}\cdot\beta m\lambda^3\right]$$

$$= \frac{N}{\beta mg} - \frac{Nh\exp(-\beta mgh)}{1 - \exp(-\beta mgh)}$$

1. (2015-2016)

d. $\Omega < \sqrt{\frac{2g(m_1 + m_2)}{m_1 a}}$

$\theta = \varepsilon$

$$\ddot{\varepsilon}(m_1 a + 2m_2 a\varepsilon^2) + \dot{\varepsilon}^2(2m_2 a\varepsilon(1 - {}^{\varepsilon^2}/_2)) + \varepsilon(-m_1 a\Omega^2(1 - {}^{\varepsilon^2}/_2) + 2g(m_1 + m_2)) = 0$$

Dropping ε^2 terms

$$\ddot{\varepsilon}(m_1 a) = -\varepsilon(2g(m_1 + m_2) - m_1 a\Omega^2)$$

$$\omega^2 = \sqrt{\frac{2g(m_1 + m_2) - m_1 a\Omega^2}{m_1 a}}$$

For $\Omega > \sqrt{\frac{2g(m_1 + m_2)}{m_1 a}}$ $\qquad \theta_0 = \cos^{-1}\left({}^{2g(m_1 + m_2)}/_{m_1 a\Omega^2}\right)$

$$\ddot{\varepsilon}(m_1 a + 2m_2 a(\sin\theta_0 + \varepsilon\cos\theta_0)) = -(\sin\theta_0 + \varepsilon\cos\theta_0)(-m_1 a\Omega^2(\cos\theta_0 - \varepsilon\sin\theta_0) + 2g(m_1 + m_2))$$

$$\ddot{\varepsilon}(m_1 a + 2m_2 a\sin\theta_0) = \sin\theta_0\, m_1 a\Omega^2\cos\theta_0 - 2g(m_1 + m_2)\sin\theta$$
$$- \sin\theta_0\, m_1 a\Omega^2\sin\theta_0\,\varepsilon + \varepsilon\cos^2\theta\, m_1 a\Omega^2 - \cancel{\varepsilon^2\cos\theta_0\sin\theta_0\, m_1 a\Omega^2}$$
$$- 2\varepsilon g(m_1 + m_2)\cos\theta_0$$
$$= \sin\theta_0\cos\theta_0\cdot m_1 a\Omega^2 - 2g(m_1 + m_2)\sin\theta_0$$
$$- \varepsilon(\sin^2\theta_0\, m_1 a\Omega^2 - \cos^2\theta_0\, m_1 a\Omega^2 + 2g(m_1 + m_2)\cos\theta_0)$$

1. (2014-2015)

c. $T = \begin{pmatrix} m_1+m_2 & m_2 l \\ m_2 l & m_2 l^2 \end{pmatrix}$ $\quad V = \begin{pmatrix} 0 & 0 \\ 0 & m_2 g l \end{pmatrix}$

$$\det(V-\lambda T) = \begin{pmatrix} -(m_1+m_2)\lambda & -m_2 l\lambda \\ -m_2 l\lambda & m_2 g l - m_2 l^2\lambda \end{pmatrix}$$

$$= -(m_1+m_2)\lambda(m_2 g l - m_2 l^2\lambda) - m_2^2 l^2\lambda^2 = 0$$

$$\lambda(m_1+m_2)(m_2 g l - m_2 l^2\lambda) + m_2^2 l^2\lambda^2 = 0$$

$$\lambda(m_1 m_2 g l - m_1 m_2 l^2\lambda + m_2^2 g l - \cancel{m_2^2 l^2\lambda}) + \cancel{m_2^2 l^2\lambda^2} = 0$$

$$\lambda = 0, \frac{(m_1+m_2)g}{m_1 l}$$

3. $$\frac{1}{r^2}\cdot\frac{d}{d\theta}\left(\frac{l^2}{mr^2}\cdot\frac{dr}{d\theta}\right) - \frac{l^2}{mr^3} = f(r)$$

$$r = k\exp(\alpha\phi)$$

$$\frac{dr}{d\theta} = \alpha k\exp(\alpha\phi)$$

$$\frac{d}{d\theta}\left(\frac{l^2\cdot\alpha k\exp(\alpha\phi)}{k^2\exp(2\alpha\phi)}\right) = \frac{d}{d\theta}\left(\frac{l^2\alpha}{k}\exp(-\alpha\phi)\right)$$

$$= \frac{l^2\alpha(-\alpha)}{k\exp(-\alpha\phi)} = \frac{-\alpha^2 l^2}{r}$$

$$\frac{-\alpha^2 l^2}{mr^3} - \frac{l^2}{mr^3} = \frac{-l^2(1+\alpha^2)}{mr^3} = f(r)$$

4. (2013-2014)

a. $$\Phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x$$

$$\rho(\vec{x}') = q\delta(x-a/2)\delta(y-a/2) + q\delta(x+a/2)\delta(y-a/2) - q\delta(x-a/2)\delta(y+a/2) - q\delta(x+a/2)\delta(y+a/2)$$

$$\Phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{[(x-a/2)^2+(y-a/2)^2+z^2]^{1/2}} + \frac{q}{[(x+a/2)^2+(y-a/2)^2+z^2]^{1/2}} - \frac{q}{[(x-a/2)^2+(y+a/2)^2+z^2]^{1/2}} - \frac{q}{[(x+a/2)^2+(y+a/2)^2+z^2]^{1/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(R^2-ax-ay+a^2/2)^{1/2}} + \frac{1}{(R^2+ax-ay+a^2/2)^{1/2}} - \frac{1}{(R^2-ax+ay+a^2/2)^{1/2}} - \frac{1}{(R^2+ax+ay+a^2/2)^{1/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[\left(1+\left(\frac{-ax-ay+a^2/2}{R^2}\right)\right)^{-1/2} + \left(1+\left(\frac{ax-ay+a^2/2}{R^2}\right)\right)^{-1/2} - \left(1+\left(\frac{-ax+ay+a^2/2}{R^2}\right)\right)^{-1/2} - \left(1+\left(\frac{ax+ay+a^2/2}{R^2}\right)\right)^{-1/2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[\frac{ax+ay-a^2/2}{2R^2} + \frac{-\cancel{ax}+ay-\cancel{a^2/2}}{2R^2} - \frac{\cancel{ax}-ay-\cancel{a^2/2}}{2R^2} - \frac{-\cancel{ax}-ay-\cancel{a^2/2}}{2R^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[\frac{4ay}{2R^2}\right] = \frac{qy}{2\pi\varepsilon_0 R^2}\left[\frac{a}{R}\right]$$

which is what we would get if we had solved

b. $$\rho(\vec{x}) = -q\,\delta(x-a/2)\,\delta(y-a/2) + q\,\delta(x+a/2)\,\delta(y-a/2)$$
$$+ q\,\delta(x-a/2)\,\delta(y+a/2) - q\,\delta(x+a/2)\,\delta(y+a/2)$$

$$\Phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{\left[(x-a/2)^2+(y-a/2)^2+z^2\right]^{1/2}} + \frac{q}{\left[(x+a/2)^2+(y-a/2)^2+z^2\right]^{1/2}}\right.$$
$$\left. + \frac{q}{\left[(x-a/2)^2+(y+a/2)^2+z^2\right]^{1/2}} - \frac{q}{\left[(x+a/2)^2+(y+a/2)^2+z^2\right]^{1/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[-\left(\frac{-ax-ay+a^2/2}{R^2}\right)^{-1/2} + \left(\frac{ax-ay+a^2/2}{R^2}\right)^{-1/2}\right.$$
$$\left. + \left(\frac{-ax+ay+a^2/2}{R^2}\right)^{-1/2} - \left(\frac{ax+ay+a^2/2}{R^2}\right)^{-1/2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\Big[$$

Will need higher orders of binomial approximation